PROBLEM OF COSMOLOGICAL SINGULARITY AND INFLATIONARY COSMOLOGY

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Abstract. Problem of cosmological singularity of general relativity theory is discussed. The possible resolution of this problem in the framework of inflationary cosmology is proposed. Physical conditions leading to bouncing inflationary solutions in the frame of general relativity theory and gauge theories of gravitation are compared. It is shown that gauge theories of gravitation allow to build regular inflationary cosmological models of closed, open and flat type with dominating ultrarelativistic matter at a bounce.

1 Problem of cosmological singularity and vacuum gravitational repulsion effect

Problem of cosmological singularity (PCS) is one of the most principal problems of general relativity theory (GR). According to Hawking-Penrose theorems the appearance of cosmological singularity in cosmological solutions of GR is inevitable, if gravitating matter satisfies so-called energy dominance conditions [1, 2]. There were many attempts to resolve PCS. If the Planckian (pre-Planckian) epoch took place in the beginning of cosmological expansion, consequent quantum theory of gravitation is necessary to analyze the PCS[1]. But such theory does not exist at present. By supposing that the Planckian epoch was absent by evolution of the Universe, classical theory of gravitation (with quantum description of gravitating mat-

[1]Recently the PCS was discussed in the frame of string theory (see [3, 4] and refs given here).

ter)can be used by analysis of the PCS. There were many such attempts to resolve the PCS corresponding to the following two approches (see [5] and refs given here).

In the frame of the first approach one supposes that energy dominance conditions are valid for gravitating matter with extremely high energy densities and pressures at the beginning of cosmological expansion. In this case GR must be replaced by certain other gravitational theory by description of gravitating matter at such extreme states. Gauge theories of gravitation (GTG) and at first of all the Poincare GTG are an important generalization of GR. Cosmological equations for homogeneous isotropic models deduced in the frame of GTG lead to regular in metrics cosmological solutions in the case of usual gravitating matter [6]. These solutions differ from corresponding solutions of GR only at extreme states of gravitating matter near certain limiting energy density, where gravitational interaction has the character of repulsion instead of attraction.

The second approach to resolve PCS concerns the analysis of description of gravitating matter at extreme conditions and examination of energy dominance conditions for corresponding states[2]. Unified gauge theories of strong and electroweak interactions with spontaneous symmetry breaking are the modern base of matter description at extreme conditions. Inflationary cosmology as important part of the theory of early Universe was built by using gauge theories of elementary particles [8,9]. A number of problems of standard Friedmann cosmology were resolved in the frame of inflationary cosmology. Most inflationary cosmological models discussed in literature are singular and their study is given from Planckian time. The radical idea of quantum birth of the Universe was introduced in order to avoid the PSC. Note that during quasi-de-Sitter inflationary stage gravitating matter in inflationary models is approximately in the state of so-called gravitating vacuum, for which pressure p and energy density $\rho > 0$ are connected in the following way $p = -\rho$ and energy dominance conditions are not valid. As it was shown in Refs. [10, 11] gravitating vacuum with sufficiently large energy density can lead to the vacuum gravitational repulsion effect (VGRE) in the case of systems including also usual gravitating matter, that allows to build regular

[2]Such analysis is important also in connection with the problem of dark matter and dark energy [7].

inflationary cosmological models. At first the VGRE was discussed in the frame of Poincare GTG [10] in the case of homogeneous isotropic models including radiation and gravitating vacuum with $\rho = \text{const}$. Because cosmological equations of Poincare GTG are valid in the frame of the most general GTG — metric-affine GTG, the VGRE can take place also in metric-affine GTG [12, 13]. Regularizing role of gravitating vacuum in GTG was analyzed in Refs [14-17]. The VGRE in GTG can lead to a bouncing solutions in the case of closed, flat and open models. It is important that the VGRE can lead to the bounce also in GR in the case of closed models [11, 5] (see also [26] and refs given here). Regular bouncing closed models filled by linear massive scalar field were studied in the frame of GR in Refs. [18, 19]. By using values of some parameters of the Universe, the probability of regular solutions for such models was estimated to be very small [18][3]. This estimation is not valid for inflationary models discussed below, for which scalar fields dominate during very small time intervals.

By using some simplest potentials for scalar fields regular inflationary cosmological models were discussed in the frame of GR as well as GTG in Refs.[22-25]. It was shown that limiting energy density and temperature at the bounce in inflationary models can be essentially smaller than the Planckian ones. Obtained regular inflationary solutions contain the stage of transition from compression to expansion and quasi-de-Sitter inflationary stage. After inflation scalar fields oscillate and are transformed into elementary particles, as result the transition to Friedmann regime takes place. Formal integration of cosmological equations leads also to quasi-de-Sitter stage of compression [22-25]. However, this stage in GR is unstable and small change of variables at compression stage can lead to singular solution. This means that the building of regular inflationary models is possible, if physical conditions leading to the bounce take place at the end of cosmological compression (see below).

By using results of numerical investigation performed in Ref.[27], the analysis of regular

[3] At the first time cosmological solution with initial de Sitter state limited in the past was discussed in Ref. [20]. The problems - what origin has initial vacuum state, what was before this state - were not considered in [20]. The cosmological solution with initial vacuum state not limited in the time in the past, proposed in Ref.[21] does not lead to resolution of PCS in the future.

inflationary models in GR and GTG in dependence on conditions at a bounce is given below in present paper.

2 Regular inflationary cosmological models in GR

Let us consider homogeneous isotropic models filled by scalar field ϕ minimally coupled with gravitation and usual matter. We suppose that interaction between them is negligibly small. Then energy density ρ and pressure p can be written in the following form

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_m, \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi) + p_m, \tag{1}$$

where a dot denotes differentiation with respect to time, index m corresponds to usual matter, and $V(\phi)$ is scalar field potential. Evolution of considered models in GR is described by Friedmann cosmological equations

$$\frac{k}{R^2} + H^2 = \frac{8\pi}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_m\right), \tag{2}$$

$$\dot{H} + H^2 = \frac{8\pi}{3M_p^2}\left(V(\phi) - \dot{\phi}^2 - \frac{1}{2}\left(\rho_m + 3p_m\right)\right), \tag{3}$$

where $k = +1, 0, -1$ for closed, flat and open models respectively, R is the scale factor, $H = \dot{R}/R$ is Hubble parameter, M_p is Planckian mass. (The system of units with $\hbar = c = 1$ is used.) By virtue of negligible interaction of scalar field with usual matter the conservation law

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \tag{4}$$

leads to equation for scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -V' \qquad \left(V' = \frac{dV}{d\phi}\right) \tag{5}$$

and

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0. \tag{6}$$

In the case of closed models (k=+1) Eqs. (2)–(3) can lead to regular bouncing solutions. Putting $t=0$ at bounce ($H(0)=0$, $\dot{H}_0 \equiv \dot{H}(0)>0$), we see bouncing solutions take place, if the value of scalar field potential $V_0 = V(\phi(0))$ (which plays the role of gravitating vacuum energy density) is sufficiently large and satisfies the following inequality

$$V_0 > \dot{\phi}_0^2 + \frac{1}{2}\left(\rho_{m0} + 3p_{m0}\right) \tag{7}$$

(index 0 corresponds to $t=0$). For definiteness we will consider later the matter in the form of ultrarelativistic matter or radiation ($\rho_m = \rho_r$, $p_m = \frac{1}{3}\rho_r$, index r corresponds to radiation), then Eq. (3) takes the following form

$$\dot{H} + H^2 = \frac{8\pi}{3M_p^2}\left(V - \dot{\phi}^2 - \rho_r\right). \tag{3$'$}$$

and according to (6) we have

$$\rho_r R^4 = \text{const.} \tag{8}$$

To obtain a bouncing solution we have to take some initial conditions for scalar field (ϕ_0, $\dot{\phi}_0$) and radiation ρ_{r0} satisfying according to (7) the following relation

$$V_0 - \dot{\phi}_0^2 - \rho_{r0} > 0 \tag{7$'$}$$

Admissible values of ϕ_0, $\dot{\phi}_0$ and ρ_{r0} must satisfy restrictions: $V_0 \lesssim 1M_p^4$, $\frac{1}{2}\dot{\phi}_0^2 \lesssim 1M_p^4$, $\rho_{r0} \lesssim 1M_p^4$, by which quantum gravitational effects are not essential and our classical consideration is valid. Minimum value of R_0 of scale factor at bounce we obtain from Eq. (2)

$$R_0 = \left[\frac{8\pi}{3M_p^2}\left(V_0 + \frac{1}{2}\dot{\phi}_0^2 + \rho_{r0}\right)\right]^{-\frac{1}{2}} \tag{9}$$

By using initial conditions at bounce we integrate Eqs. (3$'$) and (5) putting $\rho_r = \dfrac{\rho_{r0}R_0^4}{R^4}$ in accordance with (8).

Now let us discuss the most important features of regular inflationary models in GR, by taking into account numerical results obtained in the case of the simplest scalar field potentials $V_1 = \frac{1}{4}\lambda\phi^4$ ($\lambda = 10^{-14}$) and $V_2 = \frac{1}{2}m^2\phi^2$ in Ref. [25].

a) Transition stage from compression to expansion

At first we will consider bouncing solutions in the case when the value of $\eta \equiv V_0 - \dot{\phi}_0^2 - \rho_{r0} > 0$ is not near to zero. Then as numerical analysis shows, the Hubble parameter $H(t)$ and scalar field $\phi(t)$ ($\dot{\phi}_0 \neq 0$) vary in linear way during transition stage from compression to expansion, and derivatives $\dot{H}$ and $\dot{\phi}$ decrease rapidly and tend to zero at the end of this stage ($t \sim t_1$), moreover radiation energy density $\rho_r \to 0$ at $t \sim t_1$ (see Fig. 1).[4] According

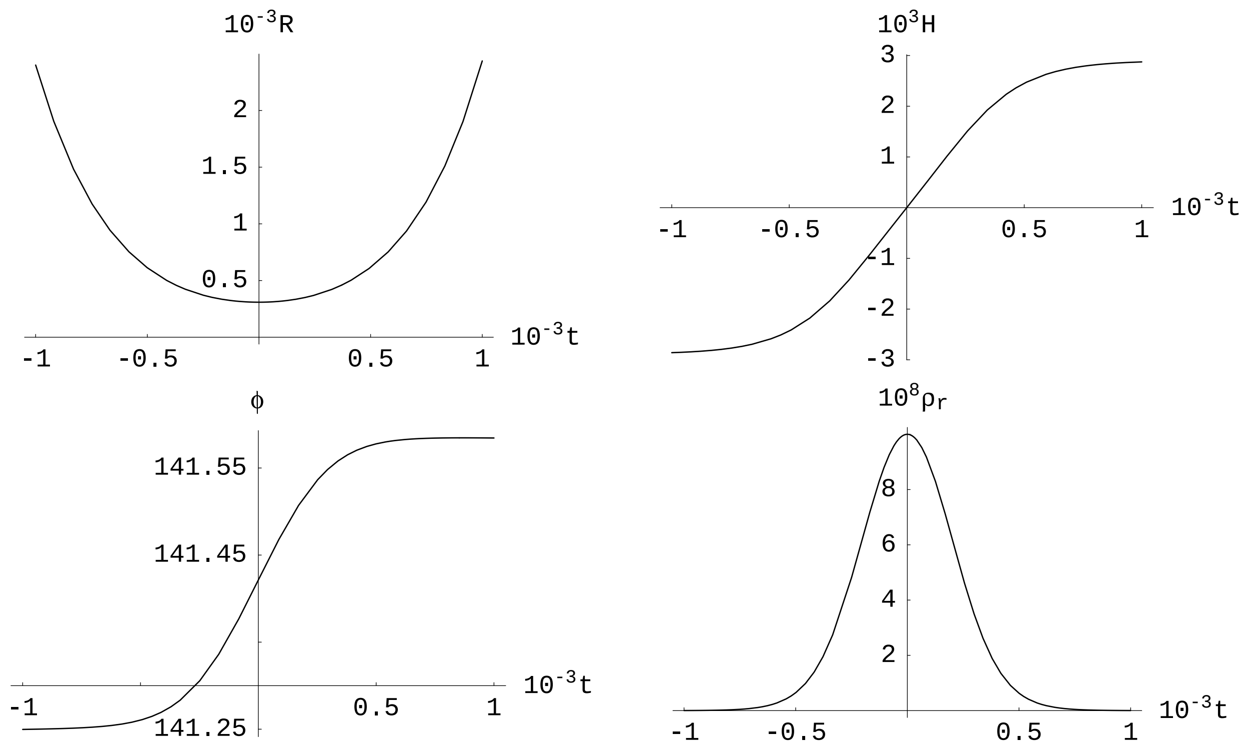

Figure 1: Dynamics of functions $R(t)$, $H(t)$, $\phi(t)$ and $\rho(t)$ during transition stage from compression to expansion in the case of initial conditions: $V_0 = 10^{-6} M_p^4$ ($\phi_0 = 141.42 M_p$), $\dot{\phi}_0 = \sqrt{0.3 V_0}$, $\rho_{r0} = 0.1 V_0$

to Eq. $(3')$ we have

$$H(t_1) = \sqrt{\frac{8\pi}{3M_p^2} V(\phi_1)} \qquad (\phi_1 = \phi(t_1)) \tag{10}$$

[4]All figures in this paper are given in system of units $\hbar = c = M_p = 1$ for the potential $V = \frac{1}{4}\lambda\phi^4$ ($\lambda = 10^{-14}$).

and the change of Hubble parameter during time interval $\Delta t_1 = t_1 - t_0$ ($t_0 = 0$, $H_0 = 0$) is

$$\Delta H_1 = H_1 = \dot{H}_0 \Delta t_1 = \frac{8\pi}{3M_p^2} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right) \Delta t_1. \tag{11}$$

Because $\phi_1 = \phi_0 + \dot{\phi}_0 \Delta t_1$ and hence according to (10) $H_1 = \sqrt{\frac{8\pi}{3M_p^2} V_0} \left(1 + \frac{1}{2} \frac{V_0' \dot{\phi}_0}{V_0} \Delta t_1 \right)$ we find from (11)

$$\Delta t_1 = \frac{\sqrt{V_0}}{\sqrt{\frac{8\pi}{3M_p^2}} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right) - \frac{1}{2} \frac{V_0'}{\sqrt{V_0}} \dot{\phi}_0} \tag{12}$$

Analogously we can determine time interval Δt_2 of transition stage before a bounce

$$\Delta t_2 = \frac{\sqrt{V_0}}{\sqrt{\frac{8\pi}{3M_p^2}} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right) + \frac{1}{2} \frac{V_0'}{\sqrt{V_0}} \dot{\phi}_0}$$

Then duration of transition stage is

$$\Delta t_{tr} = \Delta t_1 + \Delta t_2 = \frac{2 \sqrt{\frac{8\pi}{3M_p^2} V_0} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right)}{\frac{8\pi}{3M_p^2} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right)^2 - \frac{1}{4} \frac{V_0'^2}{V_0} \dot{\phi}_0^2} \tag{13}$$

and value of the Hubble parameter at the end of transition stage (at the beginning of inflationary stage) is

$$H_1 = \frac{\frac{8\pi}{3M_p^2} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right) \sqrt{V_0}}{\sqrt{\frac{8\pi}{3M_p^2}} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right) - \frac{1}{2} \frac{V_0'}{\sqrt{V_0}} \dot{\phi}_0}. \tag{14}$$

If $\frac{1}{2} \left| \frac{V_0'}{\sqrt{V_0}} \dot{\phi}_0 \right| \ll \sqrt{\frac{8\pi}{3M_p^2}} \left(V_0 - \dot{\phi}_0^2 - \rho_{r0} \right)$, we have

$$H_1 \approx \sqrt{\frac{8\pi}{3M_p^2} V_0}. \tag{15}$$

The value of H_1 (15) depends on ϕ_0 only and does not depend on $\dot{\phi}_0$ and ρ_{r0}.

By decreasing of value of η, the linear time dependence of $H(t)$ is violated and duration of transition stage increases (see Fig. 2). Formulas (13)–(14) are not applicable in this case, but expression (10) for the Hubble parameter at the end of transition stage is valid.

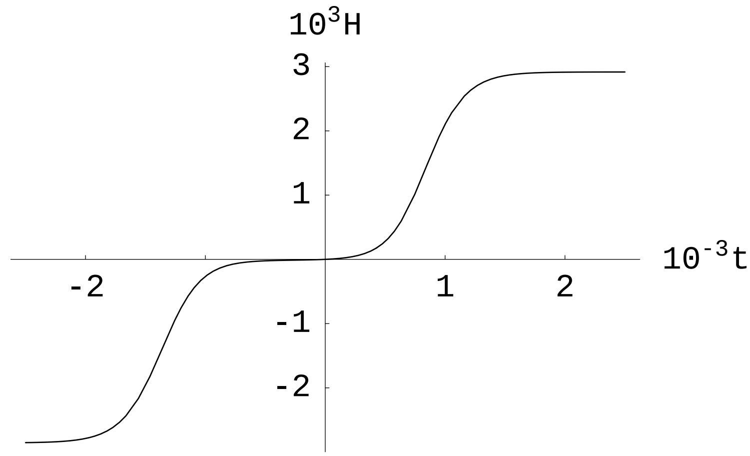

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Figure 2: Dynamics of function $H(t)$ during transition stage from compression to expansion in the case of initial conditions: $V_0 = 10^{-6}M_p^4$, $\dot{\phi}_0 = \sqrt{0.5V_0}$, $\rho_{r0} = 0.49V_0$

In the case of very small values of $\eta \to 0$ solutions change their character — inflationary stage and oscillations of scalar field vanish, solutions become unstable. For example, in the case of potential $V = \frac{1}{4}\lambda\phi^4$ ($\lambda = 10^{-14}$) we have inflationary solutions for initial conditions: a) $V_0 = 10^{-14}M_p^4$, $\dot{\phi}_0 = 0$, $\rho_{r0} = 0$; b) $V_0 = 0.1M_p^4$, $\dot{\phi}_0 = 0$, $\rho_{r0} = (1-10^{-6})V_0$; c) $V_0 = 0.1M_p^4$, $\dot{\phi}_0 = \sqrt{(1-10^{-3})V_0}$, $\rho_{r0} = 0$. However, we don't have inflationary solutions by the following change of these conditions: a) $V_0 = 10^{-15}M_p^4$ ($\phi_0 = 0.8M_p$), $\dot{\phi}_0 = 0$, $\rho_{r0} = 0$; b) $V_0 = 0.1M_p^4$, $\dot{\phi}_0 = 0$, $\rho_{r0} = (1-10^{-7})V_0$; c) $V_0 = 0.1M_p^4$, $\dot{\phi}_0 = \sqrt{(1-10^{-4})V_0}$, $\rho_{r0} = 0$.

b) Inflationary stage

During quasi-de-Sitter inflationary stage we have $\dot{H} \ll H^2$, $\dot{\phi}^2 \ll V(\phi)$, $\ddot{\phi} \ll V'$, $\rho_r \approx 0$ and term R^{-2} in Eq. (2) is negligibly small. Then according to Eq. (3′) (and Eq. (2)) the

Hubble parameter at inflationary stage is equal to

$$H=\frac{d\log R}{dt}=\sqrt{\frac{8\pi}{3M_p^2}V(\phi)} \tag{16}$$

and Eq. (5) for scalar field takes form

$$\dot{\phi}=-\sqrt{\frac{M_p^2}{6\pi}}\,\frac{d\sqrt{V}}{d\phi}. \tag{17}$$

Values of scalar field $\phi_1=\phi(t_1)$ and scale factor $R_1=R(t_1)$ play the role of initial conditions by integration of Eqs. (16)–(17) for inflationary stage. For given potential $V(\phi)$ the integration of Eq. (17) leads to the function $\phi(t)$. In the case of potential $V_1=\frac{1}{4}\lambda\phi^4$ we obtain

$$\phi(t)=\phi_1 e^{-\sqrt{\frac{\lambda}{6\pi}}M_p\,(t-t_1)} \tag{18}$$

and in the case of potential $V_2=\frac{1}{2}m^2\phi^2$ we have

$$\phi(t)=\phi_1-\frac{M_p}{2\sqrt{3\pi}}\,m\,(t-t_1)\,. \tag{19}$$

By using explicit form of scalar field ϕ we can find the scale factor by integrating Eq. (16). In the case of functions (18)–(19) we obtain

$$R(t)=R_1\exp\left[\frac{4\pi}{nM_p^2}\left(\phi_1^2-\phi^2(t)\right)\right], \tag{20}$$

where $n=4$ and $n=2$ in the case of potentials V_1 and V_2 respectively. Expressions (18)–(20) have the same form as in chaotic inflation [8], but in considered theory the values ϕ_1 and R_1 are determined by initial conditions at bounce and are not arbitrary.

At the end of inflationary stage Eqs. (16)–(17) are not valid, and behaviour of scalar field and Hubble parameter can be determined from Eqs. (3′) and (5). After inflation the Hubble parameter is small and decreases, and scalar field changes as damped oscillations (Fig. 3). Oscillations characteristics (amplitude, frequency) depend on potential parameters and practically do not depend on initial conditions at bounce. Such dependence appears in the case of bouncing solutions with small values of η near their stability boundary [27]. But

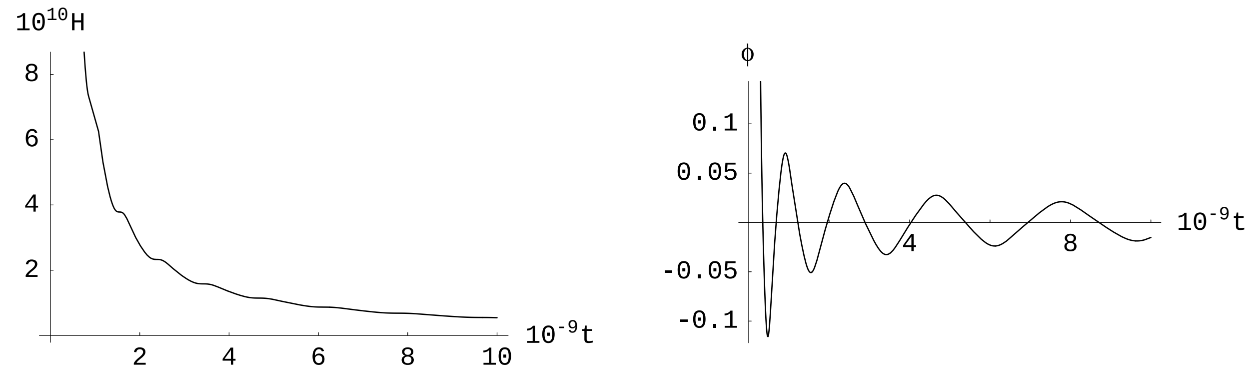

Figure 3: Dynamics of functions $H(t)$ and $\phi(t)$ after inflation ($V_0 = 10^{-6} M_p^4$, $\dot{\phi}_0 = \sqrt{0.3 V_0}$, $\rho_{r0} = 0.1 V_0$).

noted solutions do not describe inflationary models and are not interesting for inflationary cosmology. Note, that relative part of such solutions in comparison with bouncing inflationary solutions is small. As result approximately relative part of bouncing inflationary solutions on the plane (ϕ_0, $\dot{\phi}_0$) satisfying inequality ($7'$) (if $\rho_{r0} = 0$, $0 \le V(\phi_0) \le 1 M_p^4$, $0 \le \frac{1}{2}\dot{\phi}_0^2 \le 1 M_p^4$) is equal to: a)$\frac{\sqrt{2}}{6}$ for potential $V_1 = \frac{1}{4}\lambda\phi^4$, b) $\frac{\sqrt{2}}{4}$ for potential $V_2 = \frac{1}{2}m^2\phi^2$ [25].

3 Regular inflationary cosmological models in GTG

Cosmological equations for homogeneous isotropic models obtained in the frame of gauge theories of gravitation have the following form

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[R\sqrt{|1-\beta(\rho-3p)|} \right] \right\}^2 = \frac{8\pi}{3M_p^2} \frac{\rho - \frac{\beta}{4}(\rho-3p)^2}{1-\beta(\rho-3p)}, \tag{21}$$

$$\frac{\left[\dot{R} + R\left(\ln\sqrt{|1-\beta(\rho-3p)|} \right)^{\cdot} \right]^{\cdot}}{R} = -\frac{4\pi}{3M_p^2} \frac{\rho + 3p + \frac{\beta}{2}(\rho-3p)^2}{1-\beta(\rho-3p)}. \tag{22}$$

Here β is indefinite parameter with inverse dimension of energy density. At first Eqs. (21)–(22) were deduced in Poincare GTG [6], and later it was shown that Eqs. (21)–(22) take place also in metric-affine GTG [12, 13]. The conservation law has usual form (4). Besides cosmological equations (21)–(22) gravitational equations of GTG lead to the following

relation for torsion function S and nonmetricity function Q

$$S - \frac{1}{4}Q = -\frac{1}{4}\frac{d}{dt}\ln\left|1 - \beta(\rho - 3p)\right|. \tag{23}$$

In Poincare GTG $Q = 0$ and Eq. (23) determines the torsion function. In metric-affine GTG there are three kinds of models [13]: in the Riemann-Cartan space-time ($Q = 0$), in the Weyl space-time ($S = 0$), in the Weyl-Cartan space-time ($S \neq 0$, $Q \neq 0$, the function S is proportional to the function Q). Eqs. (21)–(22) coincide practically with Friedmann cosmological equations of GR and noneinsteinian characteristics S and Q are negligible if energy density is small $|\beta(\rho - 3p)| \ll 1$. The difference between GR and GTG can be essential at extremely high energy densities $|\beta(\rho - 3p)| \gtrsim 1$.[5] In the case of gravitating vacuum with constant energy density $\rho_v = \text{const} > 0$ cosmological Eqs. (21)–(22) are reduced to Friedmann cosmological equations of GR and $S = Q = 0$, this means that de Sitter solutions for metrics with vanishing torsion and nonmetricity are exact solutions of GTG [28,29] and hence inflationary models can be built in the frame of GTG [22-24]. In the case of values of β to be not large in system of units with $\hbar = c = M_p = 1$ ($\left|\beta(4V - \dot{\phi}^2)\right| \ll 1$) inflationary models of GTG practically coincide with that of GR [24]. It is interesting to investigate inflationary models in GTG in the case of large values of β ($\left|\beta(4V - \dot{\phi}^2)\right| \gg 1$).

In order to analyze inflationary cosmological models in GTG let us consider systems including scalar field and radiation, for which quantities ρ and p are given by (1), where $\rho_m = \rho_r$, $p_m = \frac{1}{3}\rho_r$. Then cosmological equations (21)–(22) by taking into account Eq. (5)

[5] Ultrarelativistic matter with equation of state $p = \frac{1}{3}\rho$ is exceptional system because Eqs. (21)–(22) are identical to Friedmann cosmological equations of GR in this case.

can be written in the following form

$$\frac{k}{R^2}Z^2+\left\{H\left[1-2\beta(2V+\dot{\phi}^2)\right]-3\beta V'\dot{\phi}\right\}^2$$
$$=\frac{8\pi}{3M_p^2}\left[\rho_r+\frac{1}{2}\dot{\phi}^2+V-\frac{1}{4}\beta\left(4V-\dot{\phi}^2\right)^2\right]Z, \tag{24}$$
$$\dot{H}\left[1-2\beta(2V+\dot{\phi}^2)\right]Z+H^2\left\{\left[1-4\beta(V-4\dot{\phi}^2)\right]Z-18\beta^2\dot{\phi}^4\right\}$$
$$+12\beta H\dot{\phi}V'\left[1-2\beta(2V+\dot{\phi}^2)\right]-3\beta\left[(V''\dot{\phi}^2-V'^2)Z+6\beta\dot{\phi}^2V'^2\right]$$
$$=\frac{8\pi}{3M_p^2}\left[V-\dot{\phi}^2-\rho_r-\frac{1}{4}\beta(4V-\dot{\phi}^2)^2\right]Z, \tag{25}$$

where $Z=1-\beta(4V-\dot{\phi}^2)$, $V'=\dfrac{dV}{d\phi}$, $V''=\dfrac{d^2V}{d\phi^2}$. Relation (23) takes the form

$$S-\frac{1}{4}Q=\frac{3\beta}{2}\,\frac{\left(H\dot{\phi}+V'\right)\dot{\phi}}{1-\beta\left(4V-\dot{\phi}^2\right)}. \tag{26}$$

Eqs.(24)-(25) lead to essential restrictions on scalar field by evolution. So in the case $k=0,+1$ from Eq.(24) follows that $Z\geq 0$, if $\beta<0$. Because at the bounce ($t=0$) we have $H(0)=0$, from (24) follows

$$\frac{k}{R_0^2}Z_0^2+9\beta^2V'^2_0\dot{\phi}_0^2=\frac{8\pi}{3M_p^2}\left[\rho_{r0}+\frac{1}{2}\dot{\phi_0}^2+V_0-\frac{1}{4}\beta\left(4V_0-\dot{\phi}_0^2\right)^2\right]Z_0 \tag{27}$$

($Z_0=1-\beta(4V_0-\dot{\phi}_0^2)$). The relation (27) determines minimum value of scale factor R_0 for closed and open models, and in the case of flat models gives the dependence between ϕ_0, $\dot{\phi}_0$ and ρ_{r0}. From (25) the time derivative $\dot{H}_0=\dot{H}(0)$ at the bounce is

$$\dot{H}_0=\left\{\frac{8\pi}{3M_p^2}\left[V_0-\dot{\phi}_0^2-\rho_{r0}-\frac{1}{4}\beta(4V_0-\dot{\phi}_0^2)^2\right]Z_0\right.$$
$$\left.+3\beta\left[(V''_0\dot{\phi}_0^2-V'^2_0)Z_0+6\beta\dot{\phi}_0^2V'^2_0\right]\right\}\left[1-2\beta(2V_0+\dot{\phi}_0^2)\right]^{-1}Z_0^{-1}. \tag{28}$$

Then the condition $\dot{H}_0>0$ determines permissible values of $\phi_0,\dot{\phi}_0,\rho_{r0}$ at the bounce in dependence on indefinite parameter β. In order to analyze the dependence $\dot{H}_0=\dot{H}_0(\beta)$ we write expression (28) in the form

$$\dot{H}_0=a\beta_1\beta_2\left(\beta^2+\frac{b}{a}\beta+\frac{f}{a}\right)(\beta-\beta_1)^{-1}(\beta-\beta_2)^{-1}, \tag{29}$$

where $\beta_1 = \frac{1}{2}(2V_0 + \dot{\phi}_0^2)^{-1}$ and $\beta_2 = (4V_0 - \dot{\phi}_0^2)^{-1}$ are particular points and

$$
\begin{aligned}
a =& \frac{2\pi}{3M_p^2}\left(4V_0 - \dot{\phi}_0^2\right)^3 + 3\left(V_0'^2 - V_0''\dot{\phi}_0^2\right)\left(4V_0 - \dot{\phi}_0^2\right) + 18V_0'^2\dot{\phi}_0^2,\\
b =& \frac{2\pi}{3M_p^2}\left(4V_0 - \dot{\phi}_0^2\right)\left(4\rho_{r0} + 5\dot{\phi}_0^2 - 8V_0\right) - 3\left(V_0'^2 - V_0''\dot{\phi}_0^2\right),\\
f =& \frac{8\pi}{3M_p^2}\left(V_0 - \dot{\phi}_0^2 - \rho_{r0}\right).
\end{aligned}
$$

If $b^2 - 4af \geq 0$, the formula (29) takes the form

$$\dot{H}_0 = a\beta_1\beta_2\left(\beta - \beta_1^{(0)}\right)\left(\beta - \beta_2^{(0)}\right)(\beta - \beta_1)^{-1}(\beta - \beta_2)^{-1},$$

where $\beta_{1,2}^{(0)} = \dfrac{-b \pm \sqrt{b^2 - 4af}}{2a}$. In the case of large (in module) values of β ($|\beta\left(4V_0 - \dot{\phi}_0^2\right)| \gg 1$) we have $\dot{H}_0 \approx a\beta_1\beta_2$ and the bounce condition $\dot{H}_0 > 0$ leads to the following relation

$$\frac{2\pi}{3M_p^2}\left(4V_0 - \dot{\phi}_0^2\right)^2 + 3\left(V_0'^2 - V_0''\dot{\phi}_0^2\right) + \frac{18V_0'^2\dot{\phi}_0^2}{\left(4V_0 - \dot{\phi}_0^2\right)} > 0 \tag{29'}$$

The relation (29′) is compatible with the following condition

$$4V_0 - \dot{\phi}_0^2 > 0, \tag{30}$$

which follows from discussed above restriction on the value of Z in the case large (in module) negative value of β ($\beta < 0$). Corresponding inflationary models are regular in metrics, Hubble parameter and torsion (nonmetricity). (Note that according to Eq. (24) the right-hand part of Eq. (26) tends to $\frac{1}{2}H$ at $Z \to 0$). Unlike the condition (7′) of GR, inequality (30) does not include radiation energy density ρ_r. In contrast to GR, the appearance of VGRE in GTG (with large negative β) does not depend on radiation, although the contribution of ultrarelativistic matter to energy density of the Universe at the bounce can be essentially greater than contribution of scalar fields.

Let us consider cosmological equations of GTG (24)–(26) in approximation of large negative β ($\left|\beta\left(4V - \dot{\phi}^2\right)\right| \gg 1$), by supposing that $\rho_r + \frac{1}{2}\dot{\phi}^2 + V \ll |\beta|\left(4V - \dot{\phi}^2\right)^2$.[6] We have

[6]This assumption does not exclude that radiation energy density can dominate at the bounce $\rho_r \gg V + \frac{1}{2}\dot{\phi}^2$.

$$\frac{k}{R^2}+\frac{\left[2H\left(2V+\dot{\phi}^2\right)+3V'\dot{\phi}\right]^2}{\left(4V-\dot{\phi}^2\right)^2}=\frac{2\pi}{3M_p^2}\left(4V-\dot{\phi}^2\right), \tag{31}$$

$$\begin{aligned}\dot{H}\left(2V+\dot{\phi}^2\right)\left(4V-\dot{\phi}^2\right)+H^2\left(8V^2-34V\dot{\phi}^2-\dot{\phi}^4\right)-12HV'\dot{\phi}\left(2V+\dot{\phi}^2\right)\\+\frac{3}{2}\left(V''\dot{\phi}^2-V'^2\right)\left(4V-\dot{\phi}^2\right)-9V'^2\dot{\phi}^2=\frac{\pi}{3M_p^2}\left(4V-\dot{\phi}^2\right)^3,\end{aligned} \tag{32}$$

$$S-\frac{1}{4}Q_1=-\frac{3}{2}\frac{V'+H\dot{\phi}}{4V-\dot{\phi}^2}\dot{\phi}\,. \tag{33}$$

Eqs. (31)–(33) do not include radiation energy density, which is not essential for dynamics of regular inflationary models in considered case. According to Eq. (31) the scale factor at the bounce R_0 is determined from

$$\frac{k}{R_0^2}+9\left(\frac{V_0'\dot{\phi}_0}{4V_0-\dot{\phi}_0^2}\right)^2=\frac{2\pi}{3M_p^2}\left(4V_0-\dot{\phi}_0^2\right). \tag{34}$$

The value of R_0 determined by (34) coincides with that of GR (formula (9)) only if $\rho_{r0}=0$ and $\dot{\phi}_0=0$. According to Eq. (32) we have

$$\begin{aligned}\dot{H}_0=&\left[\frac{\pi}{3M_p^2}\left(4V_0-\dot{\phi}_0^2\right)^3+\frac{3}{2}\left(V_0'^2-V_0''\dot{\phi}_0^2\right)\left(4V_0-\dot{\phi}_0^2\right)+9V_0'^2\dot{\phi}_0^2\right]\\&\times\left(2V_0+\dot{\phi}_0^2\right)^{-1}\left(4V_0-\dot{\phi}_0^2\right)^{-1},\end{aligned} \tag{35}$$

that corresponds to (29) in the case of large values of $|\beta|$. In accordance with Eq. (34) for closed models the following inequality at a bounce takes place

$$\frac{2\pi}{3M_p^2}\left(4V_0-\dot{\phi}_0^2\right)^3-9{V_0}'^2\dot{\phi}^2>0,$$

and in the case of open and flat models we have

$$\frac{2\pi}{3M_p^2}\left(4V_0-\dot{\phi}_0^2\right)^3-9{V_0}'^2\dot{\phi}^2\leq 0.$$

Because for open and flat models the derivative $\dot{\phi}$ does not vanish, their evolution is essentially asymmetric with respect to the bounce point.

At the beginning of the inflationary stage ($t \sim t_1$) the Hubble parameter $H(t_1)$ from (32) is equal to

$$H(t_1) = \sqrt{\frac{8\pi}{3M_p^2} V_1 + \frac{3}{4}\frac{V_1'^2}{V_1}} \qquad (V_1 = V(\phi(t_1))\,. \tag{36}$$

In the case of linear time dependence of Hubble parameter during stage of transition from compression to expansion we can simply estimate the time interval Δt_1 by using relations (35)–(36) and then determine the values of scale factor $R_1 = R_0(1 + \dot{H}_0 \Delta t_1)$ and scalar field $\phi_1 = \phi_0 + \dot{\phi}_0 \Delta t_1$ at the end of transition stage (as it was made for inflationary models in GR in Section 2). Numerical analysis of regular inflationary models in GTG with large $|\beta|$ shows that the most important difference of such models in comparison with that of GR concerns the transition stage from compression to expansion and the stage after inflation. As was noted above the region of initial conditions at the bounce of regular inflationary solutions is essentially more wide in GTG than in GR (compare ($7'$) with ($29'$), (30)). In Fig. 4 the evolution of different characteristics of regular inflationary model in GTG ($\beta = -10^{20} M_p^{-4}$) is given by choosing of initial conditions, by which a bouncing solution in GR is impossible. From Fig. 4 we see that the Hubble parameter and torsion as well as scalar field oscillate after inflation. Amplitude and frequency of scalar field oscillations are smaller in comparison with that of GR (for given potential) and decrease by increasing of $|\beta|$.

Conclusion

Considered regular inflationary models can be interesting for inflationary cosmology, if the realization of physical conditions leading to a bounce can take place at the end of cosmological compression. In the frame of GR this means that the greatest part of energy density of the Universe has to be determined by scalar fields (see ($7'$)). However, in the framework of GTG the realization of the bounce is possible, when usual ultrarelativistic matter dominates at the end of cosmological compression, and scalar fields satisfy more weak restriction (relation (30)) — its kinetic energy density can be greater than the scalar field potential. Unlike GR, in the frame of GTG the elimination of cosmological singularity and realization of the

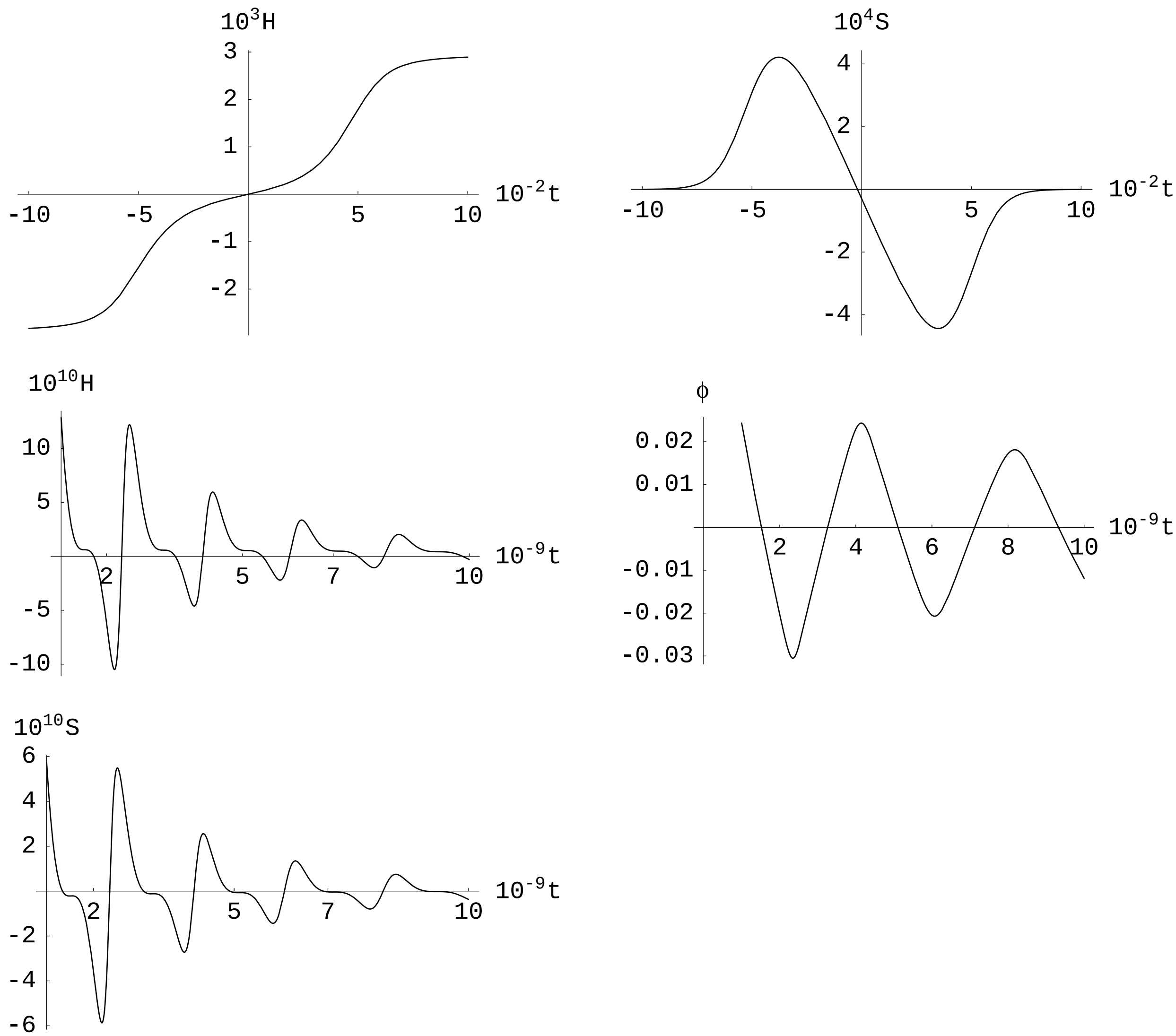

Figure 4: Dynamics of characteristics of inflationary model in GTG with $\beta = -10^{20}M_p^{-4}$ during transition stage and stage after inflation ($V_0 = 10^{-6}M_p^4$, $\dot{\phi}_0 = \sqrt{2V_0}$, $\rho_{r0} = 10^5 V_0$).

bounce in inflationary models are ensured by cosmological equations (24)-(25).

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References

[1] R. Penrose, *Structure of Space-Time.* (Mir, Moscow, 1972).

[2] S. Hawking, G. Ellis, *The Large Structure of Space-Time.* (Nauka, Moscow, 1979).

[3] M.Gasperini, G.Veneziano, *Phys. Rept 373*, 1 (2003); *hep-th/0207130.*

[4] H. Liu, G. Moore, N. Seiberg, *gr-qc/0301001.*

[5] A. V. Minkevich, *Acta Phys. Polon.* **B29**, 949 (1998).

[6] A. V. Minkevich, *Vestsi Akad. Nauk BSSR.* Ser. fiz.-mat., no. 2, 87 (1980); *Phys. Lett.* **A80**, 232 (1980).

[7] M. S. Turner, *astro-ph/0207297.*

[8] A. D. Linde, *Physics of Elementary Particles and Inflationary Cosmology.* (Nauka, Moscow, 1990).

[9] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, 1990).

[10] A. V. Minkevich, *Dokl. Akad. Nauk BSSR.* **29**, 998 (1985).

[11] A. V. Minkevich, *Vestsi Akad. Nauk Belarus.* Ser. fiz.-mat., no. 5, 123 (1995); in *Proc. of the Third Alexander Friedmann Intern. Seminar on Gravitation and Cosmology,* St.-Petersburg, July 4–12, 1995, p. 12–22.

[12] A. V. Minkevich, *Dokl. Akad. Nauk Belarus.* **37**, 33 (1993).

[13] A. V. Minkevich and A. S. Garkun, *Class. Quantum Grav.* **17**, 3045 (2000).

[14] A. V. Minkevich, N. H. Chuong, *Vestsi Akad Nauk BSSR.* Ser. fiz.-mat., no. 5, 63 (1989).

[15] A. V. Minkevich, I. M. Nemenman, *Class. Quantum Grav.* **12**, 1259 (1995).

[16] A. V. Minkevich, A. S. Garkun, *Gravitation & Cosmology.* **5**, 115 (1999).

[17] A. V. Minkevich, G. V. Vereshchagin, *Gravitation & Cosmology.* **8**, N4(24), 305 (2002).

[18] A. A. Starobinsky. *Pis'ma v Astron. Zhurn.* **4**, 155 (1978).

[19] V. A. Belinsky, L. P. Grishchuk, Ya. B. Zeldovich,I. M. Khalatnikov, *Zhurn. Exp. Teor. Fiz.* **89**, 346 (1985).

[20] E. B. Gliner, I. G. Dymnikova, *Pis'ma Astron. J.,* **24**, 235 (1975).

[21] H. J. Blome and W. Priester, *Astron. Astrophys.* **250**, 43 (1991).

[22] A. V. Minkevich, in *Nonlinear Phenomena in Complex Systems. Proceedings of the Tenth Annual Intern. Seminar NPCS-2001*, Minsk, 2001, pp. 231–236.

[23] A. V. Minkevich, G. V. Vereshchagin, ibid., p. 237–240.

[24] A. V. Minkevich, *Intern. J. Mod. Phys. A*, **17**, 4441 (2002).

[25] A. V. Minkevich, A. A. Minkevich, in *Nonlinear Phenomena in Complex Systems. Proceedings of the Eleventh Annual Intern. Seminar NPCS-2002*, Minsk, 2002, pp. 209–213.

[26] G. Ellis, R. Maartens, gr-qc/0211082.

[27] A. V. Minkevich, A. A. Minkevich, to be published.

[28] A. V. Minkevich, *Phys. Lett.* **95A**, 422 (1983).

[29] A. V. Minkevich, *Dokl. Akad. Nauk BSSR.* **30**, 311 (1986).